



Punjab Technical University

Maximum Marks: 90

Time: 90Mins.

Entrance Test for Enrollment in Ph.D Programme

Important Instructions

- ➔ Fill all the information in various columns, in Capital letters, with blue/black point pen for attempting the questions
- ➔ Use of calculators is not allowed.
- ➔ Make attempt by writing the answer in capital Letters in the box against each question number.
- ➔ All questions are compulsory. Each Question has only one right answer. No Negative marking for wrong answers.
- ➔ Questions attempted with two or more options/answers will not be evaluated.

Stream:Applied Sciences

DisciplineMathematics

Name

Fathers Name

Date

Roll Number

Signature of Candidate:

Signature of Invigilator

Q.1. Let R be a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m (i.e., $n|m$). Then, R is

- (a) reflexive and symmetric (b) transitive and symmetric
- (c) equivalence (d) reflexive, transitive but not symmetric

Q.2. If R^* is an extended real number system, then the least upper bound is R^* is

- (a) $+\infty$ (b) $-\infty$
- (c) 0 (d) no least upper bound

Q.3. Which of the following(s) is / are correct ?

- (a) The set of real number is $R \cap Q$ is countable

(b) The set of real number is $\mathbb{R} \cap \mathbb{Q}^c$ is countable

(c) Both (a) and (b)

(d) Neither (a) nor (b)

Q.4. If E is a measurable set of finite measure, $\langle f_n \rangle$, a sequence of measurable functions defined on E .

If f is a real-valued function such that for each $x \in E, f_n(x) \rightarrow f(x)$, then

(a) Given $\varepsilon > 0$ and $\delta > 0$, there is a measurable set $A \subset E$ with $mA < \delta$

(b) Given $\varepsilon > 0$ and $\delta > 0$, there does not exist a measurable set $A \subset E$ with $mA < \delta$

(c) Both (a) and (b)

(d) Neither (a) nor (b)

Q.5. A continuous function

(a) is always a function of bounded variation

(b) is never a function of bounded variation

(c) may or may not be a function of bounded variation

(d) None of the above

Q.6. Select the incorrect statement.

(a) If f is of bounded variation on $[a, b]$ to $[c, d]$ and one-one, then f^{-1} is of bounded variation on $[c, d]$, provided f^{-1} exist

(b) If f is defined from $[a, b]$ onto itself and satisfies LMVT conditions, then f is of bounded variation

(c) If $f(x) = \int_a^x g(t) dt, \forall x \in (a, b)$, then f is of bounded variation if $g(t)$ is Riemann integrable

(d) If $g(x) \in R[a, b]$, then $g(x)$ is of bounded variation on $[a, b]$

Q.7. Given the vectors $(1, 1, 0), (3, 1, 3)$ and $(5, 3, 3)$

(a) the vectors are linearly independent

(b) the vectors are linearly dependent

(c) $(1, 1, 0) + (3, 1, 3) + (5, 3, 3) = 0$

(d) $(1, 1, 0) + (3, 1, 3) + (5, 3, 3) \neq 0$

Q.8. If V be the vector space of all functions from \mathbf{R} to \mathbf{R} and $W = \{f : f(5)=0\}$.

(a) W is a subspace of V

(b) W is not a subspace of V

(c) W is closed under multiplication

(d) W is not closed under multiplication

Q.9. If (i) $L = M_1 + M_2$ and (ii) $M_1 \cap M_2 = (0)$ i.e., M_1 and M_2 are disjoint subspaces of L .

(a) A linear space L is a direct sum of its two subspaces M_1 and M_2

(b) A linear space L is not direct sum of its two subspaces M_1 and M_2

(c) $L = M_1 \oplus M_2$

(d) $L \neq M_1 \oplus M_2$

Q.10. If $\gamma : [a, b] \rightarrow \mathcal{C}$ and P is any partition of $[a, b]$. Then, total variation of γ , $V(\gamma)$ is equal to

(a) $\text{Inf} \{v(\gamma; P)\}$

(b) $\text{Sup}\{v(\gamma; P)\}$

(c) $\max \{v(\gamma; P)\}$

(d) $\min\{v(\gamma; P)\}$

Q.11. If f is analytic in a ball $B(a; R)$ and $|f(z)| \leq M, \forall z \in B(a; R)$, then

(a) $|f^{(n)}(a)| \leq \frac{n!M}{R^n}$

(b) $|f^{(n)}(a)| \geq \frac{n!M}{R^n}$

(c) $|f^{(n)}(a)| = n!M$

(d) None of these

Q.12. The series

$$\sum_{n=0}^{\infty} \frac{z^n}{n!}, |z| \leq 1$$

- $\sum_{n=1}^{\infty} \sqrt{n+1}$
- (a) uniformly but not absolutely convergent
 - (b) uniformly and absolutely convergent
 - (c) absolutely convergent but not uniformly convergent
 - (d) convergent but not uniformly

Q.13. For the function $f(z) = \sin \left[\frac{x}{|z|^2} - \frac{iy}{|z|^2} \right]$ choose the correct answer.

- (a) $f(z)$ has no singularity
- (b) $f(z)$ has finite number of singularity with exactly one
- (c) all the singularities of $f(z)$ are pole
- (d) infinity is simple pole

Q.14. Suppose that a function f is continuous in a domain D , then among the following statements

- i. f has primitive in D
- ii. The integral of $f(z)$ along any path lying in D between any two fixed points in D is independent of path
- iii. The integral of $f(z)$ along every closed contour in D is zero.

Codes

- (a) I implies III but not II
- (b) II implies III but not implied by I
- (c) I implies II and III but not implied by either of II or III
- (d) All the statements are equivalent

Q.15. If $\phi(z) = \operatorname{Re}(z) + f(z)$, where $f(z)$ is meromorphic. Then,

- (a) $\phi(z)$ is meromorphic
- (b) $\phi(z)$ and $f(z)$ has same number of singularities
- (c) $\phi(z)$ is analytic in every closed and bounded region provided it has no poles
- (d) $\phi(z)$ has no singularities

Q.16. If $(a, 7) = 1$, then $a^{12} - 1$ is divisible by k , where k is

- (a) 3
- (b) 5
- (c) 7
- (d) 8

Q.17. If integers $a, b > 1$; then the set of all integer of the form $ma + nb (m, n \in \mathbb{Z})$ includes

- (a) Both their gcd and lcm
- (b) Their gcd but not lcm
- (c) Their lcm but not gcd
- (d) Neither their lcm nor gcd

Q.18. Which of the following statement(s) is/are true ?

- (a) Every integer greater than 1 has a prime factor
- (b) Every integer greater than 1 has no prime factor
- (c) Both (a) and (b)
- (d) None of the above

Q.19. Let $(\mathbb{Z}, +)$ denote the group of all integers under addition, then the number of all automorphisms of $(\mathbb{Z}, +)$ is

- (a) 1 (b) 2 (c) 3 (d) 4

Q.20. Two permutations in the symmetric group S_n are conjugate iff

- (a) they fix the same number of elements
- (b) they have the same cycle decomposition
- (c) they have the same order
- (d) They are odd powers of each other

Q.21. If H is a subgroup of G ,

- (a) $H^{-1} = H$
- (b) $h \in H \Rightarrow h^{-1} \in H$
- (c) $H^{-1} \neq H$
- (d) $h^{-1} \in H^{-1}$ then $h \in H$

Q.22. If G is an abelian Group, then which of the following(s) is/are hold/s?

- (a) $(ab)^{-1} = a^{-1} b^{-1}, \forall a, b \in G.$
- (b) $(ab)^n = a^n b^n, \forall a, b \in G$ for any three consecutive integers $n.$
- (c) $(bab^{-1})^n = ba^n b^{-1}, \forall a, b \in G$ and $n \in \mathbb{Z}.$
- (c) Number of elements in a group G is less than or equal to four.

- Q.23. Let S_n be the symmetric group on n symbols and A_n be the alternating Subgroup of S_n , then
- A_n is normal subgroup of S_n
 - $\{S_n : A_n\} = 2$
 - Order of A_n is $n!$
 - A_n is the set of all odd permutations of S_n
- Q.24. Let $Z_6 = \{0, 1, 2, 3, 4, 5\}$ is a ring of integers modulo 6, then following(s) is/are true.
- Z_6 is a ring without zero divisor
 - Z_6 is a ring with zero divisor.
 - 3 is an idempotent element in Z_6 .
 - Z_6 is an integral domain
- Q.25. Let F be an field, then
- F has no zero divisors
 - F has no proper ideals
 - only ideals of F are $\{0\}$ and F itself
 - F has only proper ideals
- Q.26. A topological space X is locally connected iff
- for every open set U of X , each component of U is closed in X
 - for every closed set U of X , each component of U is open in X
 - for every open set U of X , each component of U is open in X
 - None of the above
- Q.27. A product of regular spaces is
- Hausdorff
 - disjoint
 - regular
 - None of these
- Q.28. Let X be a amonizable space. If a is an open covering of X , then there is a Collection D of subsets of X such that .
- D is a open covering of X
 - D is countably locally finite
 - D is a refinement of a
 - All of the above

Q.29. Let $\pi_1: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ be projection into the first coordinate. Then,

- (a) π_1 is continuous (b) π_1 is surjective
 (c) π_1 is open (d) π_1 is closed

Q.30. The differential equation, derived from $y = Ae^{2x} + Be^{-2x}$ have the order, where A, B are constants

- (a) 3 (b) 2
 (c) 1 (d) None of these

Q.31. Integrating factor of

$(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$ is $x^m y^n$, then

- (a) $m = -7, n = 2$ (b) $m = -7, n = 7$
 (c) $m = -7, n = 1$ (d) $m = -7, n = -2$

Q.32. Solution of $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$ is

- (a) $z = f_1 \left\{ \log \frac{y}{x} \right\} + x f_2 \left\{ \log \frac{y}{x} \right\}$
 (a) $z = f_1 \left\{ \log \frac{x}{y} \right\} + x f_2 \left\{ \log \frac{x}{y} \right\}$
 (a) $z = f_1 \left\{ \log \frac{x}{y} \right\} + x f_2 \left\{ \log \frac{y}{x} \right\}$

(d) None of these

Q.33. Solution of $(mz - ny)p + (nx - lz)q - mx$, is

- (a) $f(lx + my + nz, x^2 + y^2 + z^2) = 0$
 (b) $f(lx - my - nz, x^2 + y^2 + z^2) = 0$
 (c) $f(mx - nx + ny) = 0$
 (d) None of the above

Q.34. The degree of differential equation satisfying the relation

$$\sqrt{1+x^2} + \sqrt{1+y^2} = \lambda (x \sqrt{1+y^2} - \sqrt{1+x^2}) \text{ is}$$

- (a) 1 (b) 2
(c) 3 (d) None of these

Q.35. The eigen Value for the boundary value problem $x'' + \lambda = 0; x(0) = 0, x(\pi) = 0$ satisfy

- (a) $\lambda + \tan \lambda \pi = 0$ (b) $\sqrt{\lambda} - \tan \lambda \pi = 0$
(c) $\sqrt{\lambda} + \tan \sqrt{\lambda} \pi = 0$ (d) $\lambda + \tan \sqrt{\lambda} \pi = 0$

Q. 36. The initial value problem

- (a) $u_x + u_y = 1, u(s,s) = \sin s, 0 \leq s \leq 1$, has
(b) a unique solution
(c) no solution
(d) infinitely many solutions

Q.37. The accuracy of Simpson's rule quadrature for a step size h is

- (a) $O(h^2)$ (b) $O(h^3)$
(d) $O(h^4)$ (c) $O(h^5)$

Q.38. Expression of the function $x^3 + x^2 - 5x + 2$ in factorial notation is

- (a) $x^{(3)} + 4x^{(2)} - 3x^{(1)} + 2$
(b) $x^{(3)} - 4x^{(2)} - 3x^{(1)} + 2$
(c) $x^{(3)} - 4x^{(2)} + 3x^{(1)} + 2$
(d) $x^{(3)} + 4x^{(2)} - 3x^{(1)} - 2$

Q.39. The order of the difference equation

$$U_{n+2} + U_n + 3U_n = 0 \text{ is}$$

- (a) 1 (b) 2
(c) 3 (d) does not exist

Q.40. The Newton-Raphson method is used to find the roots of the equation $x^2 - 2 = 0$. If the iterations are started from -1, the iteration will

- (a) Converge to -1 (b) converge to $\sqrt{2}$
(c) Converge to $-\sqrt{2}$ (d) not converge

Q.41. Choose correct

- (a) $\nabla = 1 - E^{-1}$ (b) $\nabla = 1 + E^{-1}$
 (c) $\nabla = -1 + E^{-1}$ (d) $\nabla = 1 - E^{-1}$

Q.42. Solution of integral equation

$$y(x) = \lambda \int_a^b k(x, t) y(t) dt + F(x) \text{ is}$$

(a) $y(x) \lambda \sum_{n=1}^n C_n f_n(x) + f(x) = \pi r^2$

(b) $y(x) \lambda \sum_{n=1}^n C_n f_n(x) + f(x)$

(c) $y(x) \lambda \sum_{n=1}^m C_n f_n(x) + f(x)$

(d) $y(x) \lambda \sum_{n=1}^m f_n(x) + C_n$

Q.43. Euler's equation for the functional

$$\int_{x_1}^{x_2} \{a(x)y'^2 + 2b(x)yy' + c(x)y^2\} dx$$

- (a) first order linear differential equation
 (b) second order linear differential equation
 (c) second order non-linear differential equation
 (d) a linear differential of order more than two

Q.44. The value of a for which the integral equation

$$u(x) = a \int_a^b e^{x-1}(t) dt \text{ has trivial solution}$$

- (a) -2 (b) -1 (c) 1 (d) 2

Q.45. Non-holonomic constraints are

- (a) the constraints that can be expressed as equation form
 (b) the constraints that cannot be expressed as equation form
 (c) Equation of constraints that contain time as explicit variable
 (d) Equation of constraints that does not contain time as explicit variable.

Q.46. Lagrange's equations for a conservative holonomic dynamical system are

(a) $\left(\frac{\partial T}{\partial q_k}\right) = \left(\frac{\partial L}{\partial q_k}\right)$ ($k = 1, \dots, n$)

(b) $\left(\frac{\partial T}{\partial q_k}\right)^2 = \frac{\partial L}{\partial q_k}$ ($k = 1, \dots, n$)

(c) $\frac{d}{dt} \left(\frac{\partial T}{\partial q_k}\right) = \frac{\partial L}{\partial q_k}$ ($k = 1, \dots, n$)

(d) None of these

Q.47. Lagrangian is defined as

(a) $L = T - V$, where T is kinetic energy and V Potential energy

(b) $L = TV$, Where T is kinetic energy and V potential energy

(c) $L = T \pm V$, where T is kinetic energy and V potential energy

(d) $L = T/V$, where T is Kinetic energy and V potential energy

Q.48. The dimensions of generalized momentum

(a) are always those of linear momentum

(b) are always those of angular momentum

(c) may be those of linear momentum

(d) may be those of angular momentum

Q.49. A particle of mass m moves in a force field of potential V , then in spherical polar coordinates

(a) the kinetic energies $\frac{1}{2} m(r^2 + r^2\dot{\theta}^2 + r^2\dot{\phi}^2 \sin^2\theta)$

(b) one of the Hamilton equation of motion is $r = \frac{1}{2}$

(c) one of the Hamilton equation of motion is $\theta = \frac{Pr_\theta}{mr^2}$

(d) one of the Hamilton equation of motion is $\theta = \frac{P\phi}{mr^2 \sin^2\theta}$

Q.50. In cylindrical coordinates the Hamiltonian equation of motion are

(a) $r = \frac{P_r}{m}$

(b) $\theta = \frac{P_\theta}{m}$

(a) $z = \frac{P_z}{m}$

(a) $P_r = mr$

Q.51. If there unbiased coins are tossed, find the probability of getting at least two tails

(a) $\frac{1}{2}, \frac{3}{8}$

(b) $\frac{1}{2}, \frac{5}{8}$

(c) $\frac{1}{2}, \frac{7}{8}$

(d) None of these

Q.52. A rifleman is firing at a distant target and has only 10% chance of hitting it. The number of rounds, he must fire in order to have 50% chance of hitting it at least once is

- (a) 11 (b) 9 (c) 7 (d) 5

Q.53. The function $f(x) = \frac{x+2}{25}$ for $x = 1, 2, 3, 4, 5$ can be represented as

Probability distributions function

- (a) always true (b) always false
(c) partially true (d) can't say

Q.54. In binomial distribution the variance σ^2 and mean μ are related by

- (a) $\sigma^2 = q\mu$ (b) $\sigma^2 = \frac{\mu}{q}$
(c) $q^2\sigma^2 = \mu$ (d) None of these

Q.55. Given, a probability density function

$$f(x) = \begin{cases} r^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find the probability $P(1 \leq x \leq 2)$

- (a) $e^{-1} + e^{-1}$ (b) e^{-2} (c) e^{-1} (d) $e^{-1} - e^{-2}$

Q.56. A coin is tossed $m+n$ times, with $m > n$. The probability of getting m consecutive heads is

- (a) $\frac{(n+2)}{e^{m+1}}$ (b) $\frac{n}{e^{m+1}}$
(c) $\frac{1+n}{2^{m+1}}$ (d) None of these

Q.57. If X_1 and X_2 are independent standard normal variates, then the distribution of $\frac{y_1^2}{y_2^2}$ is

- (a) Standard normal
(b) student's t
(c) Cauchy
(d) F- distribution with (1, 1) degrees of freedom

- Q.58. The chi-square goodness of fit is based on
- multinomial distribution
 - hyper geometric distribution
 - the assumption that the character under study is normal
 - None of the above.
- Q.59 Which of the following statement is not true?
- In a symmetric distribution, the means and the median are equal.
 - The first quartile is equal to the twenty-fifth percentile.
 - In a symmetric distribution, the median is halfway between the first and the third quartiles.
 - The median is always greater than the mean.
- Q.60. The assignment problem is a
- non-linear programming problem
 - dynamic programming problem
 - integer linear programming problem
 - integer non-linear programming problem
- Q.61. In simplex method, the variable x_j leave the basis in some iteration. Then,
- S_j can enter the basis in the next iteration
 - x_j does not enter the basis in next iteration
 - (a) and (b) both true
 - None of the above
- Q.62. In a linear programming problem in standard form, there are six variables and four constraints. Then, the number of basic feasible solutions are
- 15
 - less than 15
 - more than 15
 - None of these
- Q.63. The LPP
- $$\text{Max } z = x_1 + 2x_2 + 3x_3 + 4x_4$$
- $$\text{Subject to } \frac{x_1}{2} + x_2 + \frac{3x_3}{2!} + 2x_4 \geq 8$$
- $x_1, x_2, x_3, x_4, \geq 0$ has
- no feasible solution
 - Unique optimal solution

- (c) An unbounded solution
- (d) None of the above

Q.64. The set $S = \{(x, y) : xy \leq 0\}$ is

- (a) convex and unbounded
- (b) convex and bounded
- (c) non-convex and bounded
- (d) non-convex and unbounded

Q.65. The LPP

$$\text{Max } Z = x_1 + 4x_2$$

$$\text{Subject to } x_1 + x_2 \geq 1$$

$$x_1 \leq 2$$

$$x_1, x_2 \geq 0$$

has

- (a) unbounded optimal solution
- (b) feasible region is unbounded
- (c) no optimal value
- (d) feasible region is closed

Q.66. If $\langle a_n \rangle$ is decreasing sequence of positive number $\sum a_n$ converge, then $\lim_{n \rightarrow \infty} n a_n$

- (a) ∞
- (b) 0
- (c) 1
- (d) may not exist

Q.67. If $f(x) = \sin x$. Then, the total variation of $f(x)$ on $\{0, 2\}$ is

- (a) ∞
- (b) 1
- (c) 2
- (d) 3

Q.68. A continuous function

- (a) is always a function of bounded variation
- (b) is never a function of bounded variation
- (c) may or may not be a function of bounded variation
- (d) None of the above

- Q.69. If f and g are of bounded variation, then following is false.
- $f+g$ is of bounded variation
 - $f-g$ is of bounded variation
 - fg is not bounded variation
 - f/g is of bounded variation
- Q.70. Given set of unit vector $\{(1,0,0), (0,1,0), (0,0,1)\}$
- This set consist independent vectors
 - This set consist of dependent vectors
 - This set span space V_3
 - This set is a basis of V_3
- Q.71. Given the vector space $C(\mathbb{R})$ of the complex number over real numbers
- set $\{1, i\}$ form a basis for C
 - 1 and i are linearly dependent
 - 1 and i are linearly independent
 - $\{1, i\}$ span $C(\mathbb{R})$
- Q.72. If $F\{x\}$ is the vector space of all polynomials in one indeterminate x over the field F , the infinite set $1, x, x^2, \dots$ is
- linearly independent
 - linearly dependent
 - Both (a) and (b)
 - Neither (a) nor (b)
- Q.73. If T is a linear transformation from a vector space V into a vector space W , then the condition for T^{-1} to be a linear transformation from W to V is
- T should be one-one
 - T should not onto
 - T should be one-one and onto
 - None of the above
- Q. 74. Let V be a vector space and T is linear operator on V . If W is a subspace of V , then W is invariant under T iff $\alpha \in W \Rightarrow$
- $T(\alpha) = 0$
 - $T(\alpha) \in W$
 - $T(\alpha) = a$
 - None of these

Q. 75. Let $T : \mathbb{C}^n \rightarrow \mathbb{C}^n$ be an linear operator having n distinct eigen values.

Then,

- (a) T is invertible.
- (b) T is invertible as well as diagonalizable.
- (c) T is not diagonalizable
- (d) T is diagonalizable

Q. 76. A real quadratic form in three variables is equivalent to the diagonal form $6y_1^2 + 3y_2^2 + 0y_3^2$. Then, the quadratic form is

- (a) positive definite
- (b) indefinite
- (c) positive semi-definite
- (d) negative

Q.77. If n is the order, r is the rank and s is the signature of a real quadratic form in n variables, then the quadratic form is negative semi-definite if

- (a) $s = r = n$
- (b) $-s = r = n$
- (c) $s = r < n$
- (b) $-s = r < n$

Q.78. Let A be a non-null matrix such that $A^k = 0$, then choose the incorrect statement

- (a) its every eigen value is zero
- (b) it is similar to a diagonal matrix
- (c) it is nilpotent matrix
- (d) None of the above

Q.79. The set of all bilinear transformations under the product of transformations forms

- (a) a semi group
- (b) an abelian group
- (c) a non-abelian group
- (d) None of these

Q.80. The fixed points of the mapping $w = (5z + 4)/(z + 5)$ are

- (a) 2, 2
- (b) 2, -2
- (c) -2, -2
- (d) -4/5, 5

Q.81. Define $T : Z_{12} \rightarrow Z_{12}$, then find the number of inner automorphisms

(d) A product of regular space is regular.

Q.88. Let X be a metrizable space. Then,

- (a) X is compact
- (b) X is limit point of compact
- (c) X is sequentially compact
- (d) None of the above

Q.89. Let X be a topological space. Then,

- (a) X is locally connected implies for every open set U of X , each component of U is open in X
- (b) if for every open set U of X , each component of U is open in X , then X is locally connected
- (c) exactly only one of above
- (d) neither (a) nor (b) is true

Q.90. Let $X = \{a, b, c\}$, $\mathfrak{T} = \{\phi, \{a\}, \{a, b\}, X\}$, then (X, \mathfrak{T}) is a

- (a) compact space
- (b) hausdorff space
- (c) connected space
- (d) disconnected space

