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## GENERALIZATION OF ZERO SET OF NON-BINARY TRIPLE ERROR CORRECTING BCH TYPE CODE

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**Abstract.** In our previous paper [6] we constructed some zero set  $\{1, 3^m + 1, 3^{2m} + 1\}$  and  $\{1, 3^m + 1, 3^{3m} + 1\}$  and proved that these set represented the zeros of triple error correcting code in non binary case. In this work, we proposed the sufficient condition for the existence of zero set  $\{1, 3^l + 1, 3^{2l} + 1\}$  of Non binary triple -error-correcting code having  $\gcd(l, n) = 1$ .

**Keywords:** BCH codes; zero set and triple error correcting code.

**2010 AMS Subject Classification:** 94A24.

### 1 Introduction

BCH codes are the generalization of the most famous code for single error correcting known as Hamming codes. BCH code are powerful because they have simultaneously reasonably high rates and reasonably good error correcting abilities. BCH codes have a very nice algebraic characterization and they permit efficient shift register encoding. However, their importance is due almost wholly to indeed that these codes have very effective decoding algorithm.

BCH codes can be represented by its zeros. The well-known zero set for the triple error correcting BCH code in binary case is  $\{1, 3, 5\}$ . Kasami [1] gave an idea that Binary BCH like code can be represented by distinct zero set than the existing one. The zero set of binary BCH like code proposed by Kasami is  $\{2^l + 1, 2^{3l} + 1, 2^{5l} + 1\}$ . Later Bracken and Hellesteth [4] proposed some other zero set  $\{1, 2^k + 1, 2^{2k} + 1\}$  and  $\{1, 2^k + 1, 2^{3k} + 1\}$  and proved that these zero set also represented BCH like triple error correcting codes. Further added Vinocha and

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## Construction of new quantum MDS codes derived from constacyclic codes

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Obtaining quantum maximum distance separable (MDS) codes from dual containing classical constacyclic codes using Hermitian construction have paved a path to undertake the challenges related to such constructions. Using the same technique, some new parameters of quantum MDS codes have been constructed here. One set of parameters obtained in this paper has achieved much larger distance than work done earlier. The remaining constructed parameters of quantum MDS codes have large minimum distance and were not explored yet.

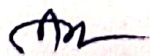
**Keywords:** Singleton bound; Hermitian construction; BCH bound; quantum MDS codes; dual containing constacyclic codes.

### 1. Introduction

Quantum error correcting codes have greatly flourished since the formation of the correlation between classical and quantum codes.<sup>1,3,4,7</sup> This has led to the construction of both binary and nonbinary linear quantum error correcting codes from classical linear codes which are self-orthogonal with respect to certain inner product.

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## A Novel approach for Segmentation of Typewritten Gurmukhi Script

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### Abstract

Demand of OCR (Optical Character Recognition) based frameworks have expanded definitely because of their noteworthy focal points into continuous applications. Segmentation assumes a significant job in optical character recognition of typewritten Gurmukhi script. Segmentation of typewritten documents is a challenging task due to the presence of skewness, overlapping, and degraded characters. Improper segmentation can hamper accuracy of character recognition. In this paper, we are proposed a new technique for line segmentation by modifying A-star algorithm and combining it with strip-based projection. Character segmentation technique is also proposed on the basis of horizontal and vertical projections combining with the aspect ratio of characters. We have accomplished accuracy of 94.28% and 99.78% with pixel count for line segmentation and 95.70% for character segmentation.

**Keywords:** OCR, Typewritten, Line segmentation, Character segmentation, Strip based projection

### 1 Introduction

OCR is one of the recent advancements in technology that has tremendously improved the hardware systems and decreased the cost of hardware implementation. Electronic means, adopt hardware-based systems for distribution and storage of data. In order to store the data, virtual and hardware-based systems have been developed which are used for storing various type of data such multimedia, documents, etc. [1].

Optical character recognition is recognition of the printed, handwritten or typewritten text by a computer.[2] This involves scanning of typewritten documents which will convert paper into an image, then translation of text image into a character code of particular script. Such implementation consists of many steps, namely digitization, pre-processing, segmentation, feature extraction and classification.[3] Segmentation is a very important step of OCR. For accomplishing segmentation, task points are required to discover from where line and character segmentation can be done. The challenge in segmentation is to discover the best focuses for segmentation. If segmentation results found inappropriate manner, it will impede the precision of recognition of framework.

Paper is considered as a primary medium for books, journals, newspapers, magazines, and business correspondence. These documents contain important information but due to a large size database, it becomes a tedious task to process the document manually for information extraction, hence any automated system is highly recommended which can be used for processing the document automatically for extracting the desired information [4]. In this work, the spotlight is on the segmentation of typewritten Gurmukhi script. Gurmukhi script is commonly used in Punjabi and it has been ranked the 14th most

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## GENERALIZED MEASURE OF FUZZY CROSS ENTROPY AND APPLICATIONS TO CODING THEORY

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### Abstract

The fuzzy cross entropy or discrimination information measure, which were introduced and proposed by Bhandari and Pal face some major draw backs as it returns undefined or meaningless value when its membership function assumes the value 0 or 1. Also, the symmetric property is not satisfied, subsequently, it cannot be used for further mathematical treatments. To overcome these shortcomings and limitations, an effort has been made by developing a new parametric generalized measure of fuzzy cross entropy. Some particular cases of the proposed cross entropy have also been studied.

### 1. Introduction

The information theory introduced by Shanon [8] measures the amount of uncertainty contained in a probabilistic experiment. The measures of

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Keywords: Membership value, Fuzzy cross entropy, coding theory, code word lengths, monotonicity.

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## MEASURE OF CROSS ENTROPY FOR DISCRETE PROBABILITY DISTRIBUTIONS

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### Abstract

As we need a modification when we have to consider a measure for the concept of distance between two probability distributions. So the concept of cross entropy is essential. In the present communication, we investigated new measure of cross entropy in probability spaces along with the graphical presentation.

### 1.1. Introduction

Information Theory' deals with a study which could be information processing, information storage, information retrieval and decision-making. Actually, 'Information Theory' studies all theoretical problems connected with the transmission of information over communication channels. The channel through which this information is transmitted may be not only the

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Keywords: convexity, directed divergence, distance measure, probability distribution.

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## EXPLORING CONSUMERS' PURCHASE INTENTION TOWARDS COSMETIC PRODUCTS IN A CHANGING LIFESTYLE LANDSCAPE IN A DEVELOPING NATION

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### ABSTRACT

*The cosmetic industry and its growing demand across consumers has been well researched in the past. However, there are several lifestyle changes encountered with the advent of social media platforms and the information sourced through its posts. The consumers today are more informed and often are seen demanding organic products when it comes to cosmetics as well. Thus, in such a changing scenario it would be interesting to note the factors that impact the purchase intention of consumers. Two types of cosmetic products are considered namely herbal and non-herbal. A total of 510 responses using a structured questionnaire have been collected from females residing in the state of Punjab in India. The responses are analyzed using ANOVA and Multiple Linear Regression. The results show that in case of herbal cosmetic products it is celebrity endorsement that motivates the purchase intention and for non-herbal products, the product packaging and perceived value has a significant impact. Moreover, the common factor motivating the purchase intention of both herbal and non-herbal cosmetic product is the body attraction provided from the use of such products.*

**Key words:** Cosmetics, Herbal, Non-Herbal, Consumers, Purchase

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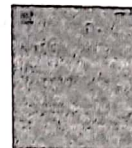
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# A tenth order $\mathcal{A}$ -stable two-step hybrid block method for solving initial value problems of ODEs

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## ARTICLE INFO

### Keywords

Ordinary differential equations  
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## ABSTRACT

In this article, a new two-step hybrid block method for the numerical integration of ordinary differential initial value systems is presented. The method is obtained after considering two intermediate points and the approximation of the true solution by an adequate polynomial and imposing collocation conditions. The proposed method has the tenth algebraic order of convergence and is  $\mathcal{A}$ -stable. The numerical experiments considered revealed the superiority of the new method for solving this kind of problems, in comparison with methods of similar characteristics appeared in the literature.

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## 1. Introduction

Differential equations arise in various fields of applications and very few of them can be solved analytically. In absence of known analytical solutions, one of the possible ways to tackle the problem is to consider a discrete domain rather than the continuous one. Then, the problem can be solved for an approximate solution by using appropriate numerical methods. Our goal is to approximate on a given interval the solution of a first-order initial value problem (I.V.P.) of the form

$$y'(x) = f(x, y(x)), \quad y(a) = y_0, \quad (1)$$

where  $x \in [a, b]$  and for now  $y, f \in \mathbb{R}$ . Further, it is assumed that the problem (1) is well-posed, that is, it has a unique continuously differentiable solution, say  $y(x)$ . Here, we denote  $y_n \approx y(x_n)$ , that is,  $y_n$  is the approximation to the theoretical solution  $y(x)$  at the nodal points  $x_n = a + nh$ ;  $n = 0, 1, 2, 3, \dots, N$ , where  $h = \frac{b-a}{N}$  is called the step-size, which may be constant or variable along the integration interval. Firstly, we will derive the block method with constant step-size and then consider a variable step-size formulation.

Although there are a huge amount of step by step methods for solving IVPs, the block methods have been developed in order to obtain the numerical solution at more than one point at a time, looking for computational efficiency. Block methods have been firstly proposed by Milne [1] to use only as a means of obtaining starting values for predictor-corrector methods. Sarafyan [2] also considered them for similar purposes. Rosser [3] developed Milne's proposals into algorithms for general purpose. For the development and use of block methods for different classes of problems see references [4–15]. The block methods contain main and additional methods, a concept that is due to Brugnano and Trigiante [7]. Some advantages

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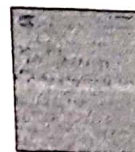
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# An efficient optimized adaptive step-size hybrid block method for integrating differential systems

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## ARTICLE INFO

**Keywords:**  
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Adaptive step-size  
Optimization strategy

## ABSTRACT

This paper deals with the development, analysis and implementation of an optimized hybrid block method having different features, for integrating numerically initial value ordinary differential systems. The hybrid nature of the proposed one-step scheme allows us to bypass the first Dahlquist's barrier on linear multi-step methods. The theory of interpolation and collocation has been used in the development of the method. We assume an appropriate polynomial representation of the theoretical solution of the problem and consider three off-step points in a one-step block. One of these three off-step points is fixed and the other two off-step points are optimized in order to minimize the local truncation errors of the main method and other additional formula. The resulting scheme is of order five having the property of  $A$ -stability. An embedded-type approach is used in order to formulate the proposed method in adaptive form, showing a high efficiency. The adaptive method is tested on well-known differential systems viz. the Robertson's system, a Gear's system, a system related with Jacobi elliptic functions, the Brusselator system, and the Van der Pol system, and compared with some well-known numerical codes in the scientific literature.

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## 1. Introduction

The mathematical formulation of many physical phenomena results in differential systems which are very difficult or even impossible to solve analytically. In such situations, differential systems are usually dealt numerically in order to find approximate solutions. This article particularly addresses the numerical solution of initial value ordinary differential systems. Conventionally, Runge-Kutta and linear multi-step methods are used in order to get numerical approximation to the theoretical solution of the problem. In these days, many numerical codes are available as built-in functions in Computer Algebra Systems (CAS), like MATLAB or MATHEMATICA, which make the task of obtaining numerical approximations to the theoretical solution accurately and efficiently. These codes are particularly designed to handle problems having different types of solutions, for example stiff, non-stiff, singular etc., in variable step-size mode.

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## An Optimized Two-Step Hybrid Block Method Formulated in Variable Step-Size Mode for Integrating $y'' = f(x, y, y')$ Numerically

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**Abstract.** An optimized two-step hybrid block method is presented for integrating general second order initial value problems numerically. The method considers two intra-step points which are selected adequately in order to optimize the local truncation errors of the main formulas for the solution and the first derivative at the final point of the block. The new proposed method is consistent, zero-stable and has seventh algebraic order of convergence. To illustrate the performance of the method, some numerical experiments are presented for solving this kind of problems, in comparison with methods of similar characteristics in the literature.

**AMS subject classifications:** 65Lxx, 65L99

**Key words:** Ordinary differential equations, second-order initial value problems, hybrid block method, optimization strategy, variable step-size.

### 1. Introduction

It is well-known that the formulation of many physical phenomena in mathematical language results in second order differential equations. For instance, the mass movement under the action of a force, problems of orbital dynamics, circuit theory, control theory, chemical kinetics, or in general, any problem involving second Newton's law.

The present article is concerned with approximating on a given interval the solution of a general second order initial value problem (I.V.P) of the form

$$y''(x) = f(x, y(x), y'(x)), \quad y(x_0) = y_0, \quad y'(x_0) = y'_0. \quad (1.1)$$

An equation of the form (1.1) can be integrated by reformulating it as a system of two first order ODEs and then applying one the methods available for solving such systems.

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# Rayleigh wave at composite porous half space saturated by two immiscible fluids

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Saturated and unsaturated media

## ABSTRACT

The propagation of Rayleigh wave along the free surface of a composite porous half-space saturated by two immiscible fluids is studied. A complex valued dispersion equation is obtained and solved numerically to examine phase speed and attenuation of Rayleigh wave propagating in the medium. Comparative study of the behavior of Rayleigh wave in saturated and unsaturated media is conducted by considering a porous rock composed of shaley sandstone and clay content with pores containing water-oil and water-gas, respectively. Variations in phase speed and attenuation of Rayleigh wave are presented with variations in porosity of medium, fluid saturation in pores and frequency parameter. Results of numerical simulation demonstrate the applicability of study of Rayleigh waves in estimating the porosity of the medium, properties of fluids and their relative saturation present in the pores.

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## 1. Introduction

The study of Rayleigh surface waves is a subject of great interest and attracts the researchers throughout the globe not only for its wide applicability but also the mathematics it involves. The Rayleigh waves travel along the boundary of the medium and their amplitudes decay rapidly with depth from surface. Understanding the behaviour of these surface waves is important in underground excavations and surface mining. The theoretical description of Rayleigh wave was firstly discussed along the plane surface of an elastic half-space by Rayleigh [1]. Existence of such wave is the result of interference of P and S waves, in which particles oscillate along an elliptical curve. After the development of the theory of wave propagation in porous medium by Biot [2,3] many researchers studied propagation of Rayleigh wave in saturated porous rocks.

In saturated porous half space Jones [4] was first to present mathematical model for propagation of Rayleigh wave by using Biot's theory. Jones [4] simplified the numerical calculations by neglecting inertial coupling between solid and fluid phases and by assuming Darcy's coefficient to be independent of frequency parameter. Also, the author considered only one out of two dilatational waves discussed in Biot's theory. Deresiewicz [5] also employed Biot's theory and studied dispersion and dissipation of Rayleigh-type surface wave in a porous, elastic half space filled with viscous liquid. Applying general boundary layer approximation to Biot's equation Mei and Foda [6], Foda and Mei [7] explained the existence of Rayleigh waves in fluid-filled half space. Tajuddin [8] extended the study of Jones [4] for a poroelastic half-space having pervious and impervious surface.

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# Effect of pore connectivity on reflection amplitudes of an inhomogeneous wave in a composite porous solid saturated by two immiscible fluids

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Present paper aims to study the phenomenon of reflection and transmission when an inhomogeneous wave strikes some discontinuity in a composite porous medium saturated by two immiscible viscous fluids. The incident wave splits into six reflected and six transmitted waves at the interface. All reflected and transmitted waves are inhomogeneous in nature with different directions of propagation vector and attenuation vector. A dimensionless parameter  $\zeta \in [0, 1]$  is introduced to represent the extent of connection among the pores at the interface. Expression of Umov–Poynting vector is derived to obtain energy flux vector. Continuity of energy flux vector at the interface gives the required boundary conditions for the system. Connecting parameter  $\zeta$  is also employed in boundary conditions to model the partial connection of pores at the interstices of two media. For numerical discussion we consider a porous medium composed of sandstone and ice, saturated with oil and water. The effect of parameter  $\zeta$  and angle of incidence is determined numerically on the amplitude and the energy ratios of reflected and transmitted waves.

**Keywords.** Composite porous medium; reflection; transmission; inhomogeneous wave; pore connectivity; energy flux.

## Nomenclature

$S_i$	Saturation of each fluid phase	$k_i$	Complex wave number of rotational wave
$S_{s_i}$	Fraction of each solid in composite matrix	$w_{1i}^f$	Normal component of drainage velocity of pore fluids
$R_{11}, R_{22}$	Coefficients related to viscous drag	$\hat{a}$	Unit normal vector to surface $S$
$A_{11}, A_{22}$	Coefficients related to inertial drag of first solid	$F, F'$	Time averaged energy flux along normal at interface in both half spaces
$B_{11}, B_{22}$	Coefficients related to inertial drag of second solid	$P_1, T_2$	Surface flow impedance for both fluids
$A_{12}$	Inertial coupling parameter connecting fluid phases	$A_m$	Attenuation vector of propagating waves
$G_{s_i}$	Shear modulus of each solid phase	$P_m$	Propagation vector of propagating waves
$k_i$	Complex wave number of dilatational wave	$A_o$	Attenuation vector of incident wave
		$P_o$	Propagation vector of incident wave
		$s$	Slowness vector of a wave
		$s_x, s_z$	Horizontal and vertical components of slowness

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Reflection and transmission of inhomogeneous waves in a  
composite porous solid saturated by two immiscible fluids

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## A mathematical model for wave propagation in a composite solid matrix containing two immiscible fluids

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**Abstract** Constitutive relations and field equations have been extended for a porous medium composed of two solids and containing two chemically non-reactive immiscible fluids. By generalizing the closure relation of porosity change and employing this into the mass balance equations, the stress-strain relations have been developed. The idea of generalized compressibility tests is invoked to find the value of dimensionless parameters appearing in the closure relation of porosity change. By generalizing momentum balance equations of Lo et al. (Water Resour Res 41:1–20, 2005), the propagation of dilatational and rotational waves is explored. It is found that four dilatational and two rotational waves exist in the porous medium. In contrast to Biot's theory, the presence of the second fluid and second solid in the porous medium gives rise to additional P- and S-waves. Variation of phase speeds and corresponding attenuation coefficients of existing waves versus frequency, saturation of the fluid phases and solid fraction are computed numerically and depicted graphically.

### 1 Introduction

Biot [1,2] developed the theory of a porous medium saturated by a viscous compressible fluid and studied wave propagation at high- and low-frequency ranges. He predicted that two compressional waves and one shear wave can travel in such a porous medium. Out of the two compressional waves, one is analogous to the dilatational wave of classical elasticity, while the second compressional wave is slower but highly attenuated. The existence of a second compressional wave was later verified experimentally by Plona [3]. Using Biot's theory, several problems of waves and vibrations have been investigated by the researchers, and they are lying in the open literature. Some notable among them are by Deresiewicz [4], Gardner [5], Deresiewicz and Skalak [6], Deresiewicz and Rice [7,8], Deresiewicz and Levy [9], Rice and Cleary [10], Yew and Jogi [11], Cleary [12], Berryman [13], De la Cruz and Spanos [14], Wu et al. [15], Albert [16], Schanz and Diebels [17] and Sharma [18]. Porous media occurring naturally or otherwise can have multiple constituents, and hence the

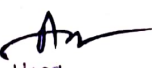
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# ON ROGERS-RAMANUJAN TYPE IDENTITIES FOR OVERPARTITIONS AND GENERALIZED LATTICE PATHS

MEGHHA GOYAL

**ABSTRACT.** In this paper we introduce and study the lattice paths for which the horizontal step is allowed at height  $h > 0$ ,  $h \in \mathbb{Z}$ . By doing so these paths generalize the heavily studied weighted lattice paths that consist of horizontal steps allowed at height zero only. Six  $q$ -series identities of Rogers-Ramanujan type are studied combinatorially using these generalized lattice paths. The results are further extended by using  $(n+t)$ -color overpartitions. Finally, we will establish that there are certain equinumerous families of  $(n+t)$ -color overpartitions and the generalized lattice paths.

## 1. Introduction and definitions

For  $n$  to be a natural number, the rising  $q$ -factorial of  $a$  with base  $q$  is defined by  $(a; q)_0 = 1$  and  $(a; q)_n = (1-a)(1-aq) \cdots (1-aq^{n-1})$ , where  $|q| < 1$ . Any series involving this rising  $q$ -factorial is called a  $q$ -series (or basic series or Eulerian series). In the literature, we see that several mathematicians have established connections between  $q$ -series, partition identities and different combinatorial parameters, see for instance [2, 9–11, 14]. Using weighted lattice paths as combinatorial tool, several basic series have been interpreted combinatorially [1, 4–6, 12]. But there are many  $q$ -series identities which cannot be interpreted combinatorially by using these classical weighted lattice paths. In this paper we will generalize these lattice paths by allowing the horizontal step to be at height  $h \geq 0$ . By doing so, the generalized lattice paths are quite helpful to interpret  $q$ -series identities combinatorially which have not been interpreted earlier in terms of weighted lattice paths. Our main objective in this paper is to use these generalized lattice paths as an elementary tool to study the following six  $q$ -series identities of Rogers-Ramanujan type combinatorially:

$$(1) \quad \sum_{\lambda=0}^{\infty} \frac{q^{\lambda^2} (-q; q^2)_{\lambda}}{(q^4; q^4)_{\lambda} (q; q^2)_{\lambda}} = \frac{(-q^2; q^{10})_{\infty} (-q^5; q^{10})_{\infty} (-q^8; q^{10})_{\infty} (-q; q^2)_{\infty}}{(q^{10}; q^{10})_{\infty}^{-1} (q^3; q^{10})_{\infty}^{-1} (q^7; q^{10})_{\infty}^{-1} (q^2; q^2)_{\infty}}.$$

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## AN ANALOGUE OF EULER'S IDENTITY AND SPLIT PERFECT PARTITIONS

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### Abstract

We give the generating function of split  $(n + t)$ -colour partitions and obtain an analogue of Euler's identity for split  $n$ -colour partitions. We derive a combinatorial relation between the number of restricted split  $n$ -colour partitions and the function  $\sigma_1(p) = \sum_{d|p} d^k$ . We introduce a new class of split perfect partitions with  $d(a)$  copies of each part  $a$  and extend the work of Agarwal and Subbarao [‘Some properties of perfect partitions’, *Indian J. Pure Appl. Math.* **22**(9) (1991), 737–743].

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*Keywords and phrases*:  $q$ -series, split  $(n + t)$ -colour partitions, perfect partitions, generating functions.

### 1. Introduction and definitions

For a natural number  $\lambda$ , the rising  $q$ -factorial of  $n$  with base  $q$  is defined by  $(a; q)_0 = 1$  and  $(a; q)_1 = (1 - a)(1 - aq) \cdots (1 - aq^{t-1})$ , where  $|q| < 1$ . Any series involving this rising  $q$ -factorial is called a  $q$ -series (or basic series or Eulerian series).

**DEFINITION 1.1.** The partition function  $p(n)$  represents the number of distinct ways of representing  $n$  as a sum of natural numbers (with order irrelevant). The generating function of  $p(n)$  is given by

$$\sum_{n=0}^{\infty} p(n)q^n = \prod_{n=1}^{\infty} \frac{1}{1 - q^n}.$$

The first result in the history of partitions is Euler's famous discovery for ordinary partitions.

**THEOREM 1.2** (Euler's identity). *The number of partitions of a positive integer  $n$  into distinct parts equals the number of partitions of  $n$  into odd parts.*

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## On $q$ -Series and Split Lattice Paths

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### Abstract

In this paper a natural question which arise to study the graphical aspect of split  $(n + t)$ -color partitions, is answered by introducing a new class of lattice paths, called split lattice paths. A direct bijection between split  $(n + t)$ -color partitions and split lattice paths is proved. This new combinatorial object is applied to give new combinatorial interpretations of two basic functions of Gordon-McIntosh. Some generalized  $q$ -series are also discussed. We further explore these paths by providing combinatorial interpretations of some Rogers-Ramanujan type identities which reveal their rich structure and great potential for further research.

**Keywords**  $q$ -series · Split  $(n + t)$ -color partitions · Split lattice paths · Combinatorial identities

**Mathematics Subject Classification** 05A15 · 05A17 · 05A19 · 11P81

### 1 Introduction

The graphical prospect of partitions and compositions has always drawn the attention of mathematicians. Graphical representation is very useful when applications of partitions and compositions are considered [9, 19, 20]. In 1987, Agarwal and Andrews [1] defined  $(n + t)$ -color partitions. To study the graphical aspect of this new set of partitions, Agarwal and Bressoud [2] introduced weighted lattice paths. Several basic series identities had been interpreted combinatorially using ordinary partitions, colored partitions, Frobenius partitions, lattice paths, associated lattice paths *etc.* in [1–5, 8, 11, 12, 14, 15, 18]. In 2014, Agarwal and Sood [6] defined a new class of partitions, called split  $(n + t)$ -color partitions. Using

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ON COMBINATORIAL EXTENSIONS OF  
ROGERS-RAMANUJAN TYPE IDENTITIES

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**ABSTRACT.** In the present paper we use anti-hook differences of Agarwal and Andrews as an elementary tool to provide new partition theoretic meanings to two generalized basic series in terms of ordinary partitions satisfying certain anti-hook difference conditions. Five particular cases are also discussed. These particular cases yield new partition theoretic versions of Göllnitz-Gordon identities and Göllnitz identity. Five  $q$ -identities of Rogers and three  $q$ -identities of Slater are further explored. These results extend the work of Goyal and Agarwal, Agarwal and Rana, and Sarcen and Rana.

## 1. INTRODUCTION AND DEFINITIONS

Several successful attempts have been made by several mathematicians to connect partition identities with other combinatorial objects such as in [3, 4, 5, 12, 13, 14]. In 1986, Agarwal and Andrews [1] introduced a new combinatorial object which they named anti-hook differences. This tool has the potential to shed new light on some fundamental classical partition identities for  $(n+t)$ -color partitions which have been introduced and studied by Agarwal and Andrews [2]. In this paper a unified combinatorial approach is made to link several colored partition identities with ordinary partitions satisfying certain anti-hook difference conditions. The results are proved by establishing bijections between appropriate classes of  $(n+t)$ -color partitions and ordinary partitions with certain anti-hook difference conditions. Five basic series identities have also been studied as the particular cases. Out of these five identities, three identities yield new partition theoretic interpretations of Göllnitz-Gordon identities and Göllnitz identity. Further, five identities of Rogers and three identities of Slater are also explored using the same technique. These new results are proved by establishing bijections between two different classes of partitions. Now before stating our main results we first recall some definitions.

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